**Homework 14**

**P23.1.2** Determine the FT of the following functions:

(a) *t*sign(*t*)

(b) |*t*| (Hint: |*t*| )

**Solution:** (a) Using the multiplication by *t* property, F{*t*sign(*t*)} = 

(b) , or |*t*| = . With , it follows from Table 23.3.1 that: F{|*t*|} =F{sgn(*t*)} =.

**P23.1.6** Determine the IFT of the following functions:

(a) 2*u*(*ω* + 2) – 2*u*(*ω* – 2).

(b) .

(c) .

**Solution:** (a) The FT of sinc(*βt*) is a rectangular pulse that extends from -*β* to +*β* and of amplitude *π*/*β.* It follows that the IFT of a rectangular pulse 2*u*(*ω* + 2) – 2*u*(*ω* – 2) is sinc(*βt*), or sinc(*2t*).

(b) ; taking the IFT gives sgn(*t*) – .

(c) ; taking the IFT gives.

**P23.1.13** Assume that the function shown in Figure 23.3.3 is in the frequency domain. Determine *f*(*t*) and verify by applying duality to the result of Example 23.3.3.

**Solution:** ;     . *f*(*t*) = . Applying duality, *F*(*jt*) is obtained by substituting *t* for *ω* in *F*(*jω*) of Example 23.3.3, which gives:  = . This is 2*πf*(-*ω*), where *f*(-*ω*) = -*f*(*ω*), because the function is odd, and *f*(*ω*) is the function of Fig. 16.3.2 in the frequency domain. Thus, the IFT of *f*(*ω*) is  , as above.

**P23.1.17** Determine *F*(*jω*) of *f*(*t*) = *A*(sin*t*)*u*(-*t*) + *A*(sin*t*)*u*(*t*) (Figure P23.1.17).

**Solution:** . From the Appendix,  . Substituting *x = t*, *a = -jω*, and *b* = 1: *F*(*jω*) . Hence, .

**P23.1.22** Determine the IFT of .

**Solution:** ; it follows that

*F* -1.

**P23.2.4** Determine *iO*(*t*) and *vO*(*t*) in Figure P23.2.4 if *vSRC* = 0.5sgn(*t*) V.

**Solution:** 

 =  . It follows that *iO* =  A. The current is zero at *t* = 0- and *t* = 0+, as expected.

  . The IFT is  V. At *t* = 0- *vO* = 0.5sgn(t) = -0.5 V.

At *t* = 0+, *vO*V, as expected for the capacitor.

**P23.3.6** If *i*(*t*) = 2sinc(4*t*), determine: (a) the frequency band starting at *ω* = 0 that contains half the energy of *i*(*t*), (b) the energy dissipated in a 2 Ω resistor due to *i*(*t*) applied over all time.

**Solution:** (a) The Fourier transform of *Im*sinc(4*t*) is . Since this is a rectangular waveform from -4 rad/s to 4 rad/s, it follows that the frequency band that contains half the energy is 0-2 rad/s.

(b) |*F*(*jw*)|2 = (2*π*/4)2 = *π2*/4. The energy dissipated in a 1 Ω resistor is (1/*π*)(*π2*/4)×4 = *π* J. The energy dissipated in 2 Ω resistor is 2*π* J.

**P23.3.7** If V in Figure P23.3.7, determine the percentage of the 1 Ω energy of *vO* in the frequency range rad/s.

**Solution:** ; ;  ; |*Vo*(*jω*)|2 =

. Referring to a Table of Integrals of the IFT, *WO*1Ω =  J. For the interval from 0 to 2 rad/s the energy is = = 49.22 J. The percentage is therefore .